TD - Calculs avec des fractions

① $A = \frac{3}{4} + \frac{4}{5} = \frac{3\times5}{4\times5} + \frac{4\times4}{5\times4} = \frac{15}{20} + \frac{16}{20} = \frac{15+16}{20} = \frac{31}{20}$

$B = \frac{1}{2} - \frac{5}{3} = \frac{1\times3}{2\times3} - \frac{5\times2}{3\times2} = \frac{3}{6} - \frac{10}{6} = \frac{-7}{6}$

$C = -\frac{15}{4} \times \frac{1}{-5} = -\frac{15\times1}{4\times(-5)} = -\frac{15}{-20} = \frac{15}{20} = \frac{3\times5}{4\times5} = \frac{3}{4}$

$D = 2 - 2 : \frac{1}{4} = 2 - 2 \times \frac{4}{1} = 2 - 2\times4 = 2 - 8 = -6$.

② $E = \frac{-3-5}{2\times3,4} = \frac{-8}{6,8} = \frac{-8\times10}{6,8\times10} = \frac{-80}{68}$ une fraction ! Rationnel non entier

$F = \frac{252}{2\times3\times7} = \frac{2\times126}{2\times3\times7} = \frac{126}{3\times7} = \frac{3\times42}{3\times7} = \frac{42}{7} = \frac{7\times6}{7\times1} = \frac{6}{1} = 6$

donc F est rationnel et entier.

$G = \left(\frac{1}{5}+3\right)^2 = \left(\frac{1}{5}+\frac{15}{5}\right)^2 = \left(\frac{16}{5}\right)^2 = \frac{16}{5}\times\frac{16}{5} = \frac{256}{25}$

donc G est rationnel mais pas entier car 25 ne divise pas 256.

$H = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{7}{8}$

donc H est rationnel mais non entier.

③ $I = \frac{13}{8} - \frac{3}{8} = \frac{10}{8} = \frac{5\times 2}{4\times 2} = \frac{5}{4}$

$J = \frac{5}{6} + \frac{1}{12} = \frac{5\times 2}{6\times 2} + \frac{1}{12} = \frac{10}{12} + \frac{1}{12} = \frac{11}{12}$

$K = \frac{-3}{5} + \frac{6}{15} = \frac{-3\times 3}{5\times 3} + \frac{6}{15} = \frac{-9}{15} + \frac{6}{15} = \frac{-3}{15}$

$L = \frac{1}{14} - \frac{3}{4} = \frac{1\times 2}{14\times 2} - \frac{3\times 7}{4\times 7} = \frac{2}{28} - \frac{21}{28} = \frac{-19}{28}$

④ $M = \frac{-4}{7} \times \frac{3}{5} = \frac{-4\times 3}{7\times 5} = \frac{-12}{35}$

$N = \frac{4}{15} \times \frac{6}{10} = \frac{4\times 6}{15\times 10} = \frac{2\times 2\times 2\times 3}{3\times 5\times 2\times 5} = \frac{2\times 2}{5\times 5} = \frac{4}{25}$

$P = \frac{-7}{4} : \frac{21}{12} = \frac{-7}{4} \times \frac{12}{21} = \frac{-7\times 2\times 2\times 3}{2\times 2\times 3\times 7} = -1$

$Q = \frac{-8}{25} : \frac{7}{15} = \frac{-8}{25} \times \frac{15}{7} = \frac{-8\times 3\times 5}{5\times 5\times 7} = \frac{-8\times 3}{5\times 5} = \frac{-24}{25}$

⑤ 1) a) $\frac{3\times 7\times 13}{7\times 11\times 13} = \frac{3}{11}$ b) $\frac{2\times 2\times 3\times 5}{3\times 5\times 7} = \frac{2\times 3}{7} = \frac{6}{7}$

c) $\frac{11\times 5\times 2^2}{2\times 7\times 11} = \frac{11\times 5\times 2\times 2}{2\times 7\times 11} = \frac{5\times 2}{7} = \frac{10}{7}$

2) $45 = 9\times 5 = 3\times 3\times 5$
$65 = 5\times 13$
donc $\frac{45}{65} = \frac{3\times 3\times 5}{5\times 13} = \frac{9}{13}$.

3) a) $-\frac{30}{36} = -\frac{2\times 3\times 5}{2\times 3\times 2\times 3} = \frac{5}{2\times 3} = \frac{5}{6}$ rationnel non entier

$\frac{25}{55} = \frac{5\times 5}{5\times 11} = \frac{5}{11}$ rationnel non entier

$\frac{28}{-14} = -\frac{2\times2\times7}{2\times7} = -\frac{2}{1} = -2$ rationnel et entier relatif

$\frac{170}{85} = \frac{2\times5\times17}{5\times17} = \frac{2\times17}{17} = \frac{34}{17}$ rationnel non entier

(6) $A = 7\times\left(\frac{4}{15} - \frac{3}{55}\right) = 7\times\left(\frac{28}{105} - \frac{6}{105}\right) = 7\times\frac{22}{105} = \frac{154}{105}$

$B = \frac{1-\frac{2}{7}}{1+\frac{2}{7}} = \frac{\frac{5}{7}}{\frac{9}{7}} = \frac{5}{7}\times\frac{7}{9} = \frac{5}{9}$

$C = \frac{6}{39} - \frac{6}{13} : \frac{9}{143} = \frac{6}{39} - \frac{6}{13}\times\frac{143}{9} = \frac{6}{3\times13} - \frac{2\times3\times143}{13\times3\times3}$

$= \frac{6-286}{3\times13} = -\frac{280}{39}$

(7) $D = \frac{\frac{5}{4}-\frac{2}{3}}{\frac{1}{6}+\frac{1}{2}} = \frac{\frac{7}{12}}{\frac{8}{12}} = \frac{7}{12}\times\frac{12}{8} = \frac{7}{8}$

$E = \frac{\frac{5}{2}+\frac{3}{4}}{1-\frac{3}{4}} = \frac{\frac{13}{4}}{\frac{1}{4}} = \frac{13}{4}\times\frac{4}{1} = 13$

$F = \frac{\frac{9}{6}\times\frac{8}{4}}{1+\frac{1}{2}} = \frac{\frac{3}{2}\times2}{\frac{3}{2}} = 2$

$G = \frac{\frac{3}{2}-\frac{7}{3}}{\frac{1}{8}+\frac{1}{24}} = \frac{\frac{-5}{6}}{\frac{4}{24}} = -\frac{5}{6}\times\frac{24}{4} = -5$

L'intru est D car c'est le seul non entier.

(3) 1) a) 0 b) $\frac{1}{5}$ c) $-\frac{3}{4}$ d) $\frac{5}{7}$

2) Le résultat est égal au nombre de départ. Pour le justifier, on peut utiliser le calcul littéral. Si on note x le nombre de départ, le résultat est :

$$\frac{2}{3}\left(x+\frac{1}{2}\right) - \frac{1}{3} + \frac{1}{3}x$$

$$= \frac{2}{3}x + \frac{2}{3}\times\frac{1}{2} - \frac{1}{3} + \frac{1}{3}x$$

$$= \frac{2}{3}x + \frac{1}{3} - \frac{1}{3} + \frac{1}{3}x$$

$$= \frac{2}{3}x + \frac{1}{3}x$$

$$= x .$$